NOTATION

Re = wd/ν, Reynolds number; Pr = ν/D, diffusion Prandtl number; φ, angle between the frontal critical point and the point of detachment of the boundary layer; w, speed of the incoming flow; d, diameter of the sphere; ν, kinematic viscosity of the liquid; D, diffusion coefficient.

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PORE MOTION AND CHANGE OF SHAPE IN A TEMPERATURE FIELD BECAUSE OF EVAPORATION AND CONDENSATION PROCESSES

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Mass transfer in a vacuum spherical and cylindrical pore due to evaporation and condensation of the wall material in a temperature gradient field is examined on the basis of a molecular—kinetic analysis.

The temperature gradient over a pore section causes transport of the wall material and results in the origination of directional pore motion. A number of mass transport mechanisms exist that cause such motion. An analysis of different motion mechanisms and conditions under which some one is dominant can be found in [1]. The governing mass transfer mechanism for large-scale vacuum pores at high temperatures is transport through the pore volume, i.e., recondensation; hence, mass transfer occurs in the free-molecule regime in a solid with low saturated vapor pressure. This regime was investigated in a vacuum pore in a number of papers [1-3], in which, however, a macroscopic description of the mass transport is used, and only the case of small temperature gradients over the pore section is examined. The kinetic approach used in this paper has a number of advantages as compared to the macroscopic description, since it permits refinement of the velocity of pore motion, the consideration of the mass transport for an arbitrarily given wall-temperature distribution (usually the case when a constant temperature gradient exists far from the pore is considered), and also nonlinear problems, and the clarification of the nature of the change in its shape.

Let us examine the mass transfer in a spherical pore and a cylindrical channel whose wall temperature is constant along the channel axis z. We shall assume that both the evaporating and the reflected molecules have a Maxwell velocity distribution, a condensation factor β independent of the temperature, and isotropic pore-surface characteristics. Under these assumptions, the expression for the mass flux density incident on the pore surface at the point r is written analogously to what is done for free-molecular flow around bodies with concavities [4]:

$$q(\mathrm{r}) = \sqrt{\frac{m}{2\pi k}}\,\beta \iint\limits_{S} \frac{P(\mathrm{r}_1)}{\sqrt{T(\mathrm{r}_1)}}\, G(\mathrm{r}\ \mathrm{r}_1)\,d\mathrm{r}_1 + (1-\beta) \iint\limits_{S} q(\mathrm{r}_1)\, G(\mathrm{r}\mathrm{r}_1)\, d\mathrm{r}_1, \tag{1}$$

where P(r) is the saturated vapor pressure of the pore wall material at the temperature T(r).

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The integration on (1) is over the whole pore surface visible from the point r. The first term on the right side of (1) is the contribution of particles being evaporated from the pore surface to the mass flux density, and the second term is from particles being reflected from the pore surface. The influence function $G(r, r_1)$ of a pore surface element dr_1 on the element $\partial_r G(r, r_1)$ is usually expressed [4] in terms of the angles θ and θ_1 between a line connecting the surface elements dr and dr_1, and their normals. For a complete diffusion nature of the reflected and evaporated particles distribution functions

$$G(\mathrm{r}, \mathrm{r}_1) = \frac{\cos\theta\cos\theta_1}{\pi r_{12}^2}, \tag{2}$$

where r_{12} is the distance between the surface elements under consideration.

For a spherical pore $\cos\theta = \cos\theta_1 = r_{12}/2R$. Therefore, the kernel of the integral equation (1) turns out to be a constant. The mass flux density incident on the wall is also constant over the whole pore surface. We obtain for the resultant mass entrainment from the wall

$$M(\mathrm{r}) = \sqrt{\frac{m}{2\pi k}}\,\beta\left[\frac{P(\mathrm{r})}{\sqrt{T(\mathrm{r})}} - \frac{1}{4\pi R^2}\iint_S \frac{P(\mathrm{r}_1)}{\sqrt{T(\mathrm{r}_1)}}\,d\mathrm{r}_1\right]. \tag{3}$$

Let us examine the case when a constant temperature gradient ∇T exists far from the pore in the solid body bulk, and the temperature change over the pore section is slight. The wall temperature distribution has the following form in this case [1]:

$$T(\chi) = T_0 + (1+\varkappa)R|\nabla T|\cos\chi, \tag{4}$$

where T_0 is the unperturbed value of the temperature at the location of the pore center, and χ is the angle between the radius-vector of the point under consideration r and ∇T.

It should be noted that the kinetic consideration of heat transmission in a pore by radiation, which is performed perfectly analogously to the mass transport consideration for light scattering by the Lambert law, will result in a different value of the coefficient $\varkappa$ from that obtained in [1]:

$$\varkappa = \frac{\lambda - 4\varepsilon R\sigma T_0^3}{2\lambda + 4\varepsilon R\sigma T_0^3}$$

(the coefficient in the term associated with heat transmission by radiation equals 4 instead of the 8/3 in [1]). Expanding $P/\sqrt{T}$ in (3) in a series and keeping the first terms of the expansion, we obtain

$$M(\chi) = \sqrt{\frac{m}{2\pi k}}\,\beta\left[(1+\varkappa)R\cos\chi|\nabla T|\frac{d}{dT}\frac{P}{\sqrt{T}}\bigg|_{T_0} + \frac{1}{2}(1+\varkappa)^2R^2\left(\cos^2\chi - \frac{1}{3}\right)(\nabla T)^2\frac{d^2}{dT^2}\frac{P}{\sqrt{T}}\bigg|_{T_0}\right]. \tag{5}$$

The first component in the right side of (5) describes the pore motion without any change in its shape. The velocity of such motion is proportional to the pore radius, hence the value of the velocity obtained in this paper is 1.5 times higher than the value obtained in [1]. The expression (5) permits estimation of the distance l traversed by the spherical pore without substantial change in its shape. Taking only the quadratic nonlinearity into account, and using the Clapeyron-Clausius equation [5]

$$P = P_0\exp\left(-\frac{Q}{kT}\right), \tag{6}$$

we obtain

$$l \ll \frac{2}{\dfrac{|\nabla T|}{T}\,\dfrac{Q}{kT}}.$$

Therefore, a pore can traverse a large distance without substantial change in its shape in the field of a weak temperature gradient. The second term in (5) describes the change in pore shape; it is seen that with the second term taken into account the pore acquires the shape of an oval extended in the direction of the temperature gradient. In this approximation the change in shape of pores of different radii as they traverse an identical distance will turn out to be identical.

Computations of the mass transport for a significant temperature drop over the pore section by using the exact expression (5) show that in this case it will point towards the hot side.

A change in pore shape in conformity with the Le Chatelier principle stimulates the process tending to weaken this change. In this case such a process is the mass transport from sections with a large radius of curvature to sections with a smaller surface radius of curvature. The saturated vapor pressure over a fluid surface having a radius of curvature R is determined by the expression [5]

$$P(R) = P(\infty)\exp\left(\frac{2\omega\gamma}{RkT}\right), \tag{7}$$

where ω is the volume per single particle in the condensed phase, and γ is the coefficient of surface tension.

This relationship is qualitatively valid even for a solid—vapor system, and can be utilized to estimate the velocity of pore spheroidization. By equating the pressure drop caused by the quadratic nonlinearity to the pressure drop associated with the change in the radius of curvature over the pore surface, we obtain an estimate of the pore deformation in the equilibrium state

$$\frac{\Delta R}{R} = R^3\frac{kT}{4\omega\gamma}\left(\frac{Q}{kT}\right)^2\left(\frac{\nabla T}{T}\right)^2.$$

The deformation turns out to be proportional to the cube of the radius. Therefore, shallow pores should remain spherical, while coarse pores can change shape significantly. The deviation from sphericity grows rapidly with the temperature gradient and the reduction of the temperature level.

Let us consider the mass transport far from the ends in a long cylindrical channel of radius R. The channel wall temperature varies in a given manner $T = T(\varphi)$. For a cylindrical channel $G(\mathbf{r}, \mathbf{r}_1)$ has the form [4]

$$G(z,\ \varphi,\ z_1\varphi_1) = \frac{4R^2\sin^4\frac{\varphi_1-\varphi}{2}}{\pi\left[(z_1-z)^2 + 2R^2\sin^2\frac{\varphi_1-\varphi}{2}\right]^2}.$$

Since the temperature is independent of the coordinate z, then by integrating with respect to z_1 in (1), we obtain the following integral equation for the mass flux density incident on the channel wall:

$$q(\varphi) = \sqrt{\frac{m}{2\pi k}}\,\frac{\beta}{4}\int\limits_{\varphi}^{\varphi+2\pi}\frac{P(\varphi_1)}{\sqrt{T(\varphi_1)}}\sin\frac{\varphi_1-\varphi}{2}\,d\varphi_1 + \frac{1-\beta}{4}\int\limits_{\varphi}^{\varphi+2\pi} q(\varphi_1)\sin\frac{\varphi_1-\varphi}{2}\,d\varphi_1. \tag{8}$$

The solution of (8) is written in the form

$$q(\varphi) = \sqrt{\frac{m}{2\pi k}}\,\frac{\sqrt{\beta}}{4}\Bigg[\Bigg[\int\limits_{0}^{2\pi}\frac{P(\varphi_1)}{\sqrt{T(\varphi_1)}}\sin\frac{\sqrt{\beta}}{2}(2\pi+\varphi-\varphi_1)\,d\varphi_1 + \tag{9}$$

$$+\frac{\sin\pi\sqrt{\beta}}{1-\cos\pi\sqrt{\beta}}\int\limits_0^{2\pi}\frac{P(\varphi_1)}{\sqrt{T(\varphi_1)}}\cos\frac{\sqrt{\beta}}{2}(2\pi+\varphi-\varphi_1)\,d\varphi_1+2\int\limits_0^{\varphi}\frac{P(\varphi_1)}{\sqrt{T(\varphi_1)}}\sin\frac{\sqrt{\beta}}{2}(\varphi-\varphi_1)\,d\varphi_1\Bigg]. \qquad (9)$$

For $\beta = 1$ Eq. (9) simplifies considerably and we have for the resultant mass flux on the wall

$$M(\varphi)=\sqrt{\frac{m}{2\pi k}}\left[\frac{P(\varphi)}{\sqrt{T(\varphi)}}-\frac{1}{4}\int\limits_{\varphi}^{\varphi+2\pi}\frac{P(\varphi_1)}{\sqrt{T(\varphi_1)}}\sin\frac{\varphi_1-\varphi}{2}\,d\varphi_1\right]. \qquad (10)$$

When a constant temperature gradient perpendicular to the cylinder axis exists far from the pore and the temperature drop over the pore section is small, then by limiting ourselves to taking account of the quadratic nonlinearity we obtain for the mass entrainment from the wall

$$M(\varphi)=\sqrt{\frac{m}{2\pi k}}\left[\frac{4}{3}(1+\varkappa)R\cos\varphi\,|\nabla T|\frac{d}{dT}\frac{P}{\sqrt{T}}\bigg|_{T_0}-\frac{4}{15}(1+\varkappa)^2R^2(1-2\cos^2\varphi)(\nabla T)^2\frac{d^2}{dT^2}\frac{P}{\sqrt{T}}\bigg|_{T_0}\right], \qquad (11)$$

where

$$\varkappa=\frac{\lambda-\dfrac{16\varepsilon}{4-\varepsilon}\sigma T_0^3R}{\lambda+\dfrac{16\varepsilon}{4-\varepsilon}\sigma T_0^3R}.$$

The first term in the right side of (11) describes pore movement without a change in its shape. The velocity of pore motion turns out to be twice the value obtained in [1]. The second term describes the change in the pore shape. In this approximation, the channel section acquires the shape of an oval extended in the direction of the temperature gradient. The expression (11) governing the mass transport in a cylindrical channel is analogous in structure to the corresponding expression for the spherical pore (5). Therefore, the estimates regarding the change in shape obtained for a spherical pore remain valid even for a cylindrical channel.

NOTATION

m, molecule mass; k, Boltzmann constant; R, radius of a spherical pore and a cylindrical channel; λ, heat conduction; ε, emissivity; σ, constant in the Stefan—Boltzmann law; Q, heat of evaporation.

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